(ii) In Y(HV)–Δ(LV) transformers, if a phase shift is included as per the American-standard notation, the ratio __________ is used in positive-sequence network, and __________ the ratio __________ is used in the negative-sequence network.

(iii) The base voltages depend on the winding connections; the per-unit impedances (a) do or (b) do not depend on the winding connections.

SECTION 8.7

8.26 In per-unit sequence models of three-phase three-winding transformers, for the general zero-sequence network, the connection between terminals H and H′ depends on how the high-voltage windings are connected:
 (i) For solidly grounded Y, __________ H to H′.
 (ii) For grounded Y through Z_n connect __________ from H to H′.
 (iii) For ungrounded Y, leave H–H′ __________.
 (iv) For Δ, __________ H′ to the reference bus.

SECTION 8.8

8.27 The total complex power delivered to a three-phase network equals (a) 1, (b) 2, or (c) 3 times the total complex power delivered to the sequence networks.

8.28 Express the complex power S_S delivered to the sequence networks in terms of sequence voltages and sequence currents, where $S_S =$ __________.

PROBLEMS

SECTION 8.1

8.1 Using the operator $a = 1\angle 120°$, evaluate the following in polar form: (a) $(a-1)/(1+a-a^2)$, (b) $(a^2+a+j)/(ja+a^2)$, (c) $(1+a)(1+a^2)$, and (d) $(a-a^2)(a^2-1)$.

8.2 Using $a = 1\angle 120°$, evaluate the following in rectangular form:

a. a^{10}

b. $(ja)^{10}$

c. $(1-a)^3$

d. e^a

Hint for (d): $e^{(x+jy)} = e^x e^{jy} = e^x \angle y$, where y is in radians.

8.3 Determine the symmetrical components of the following line currents: (a) $I_a = 6\angle 90°$, $I_b = 6\angle 320°$, $I_c = 6\angle 220°$ A and (b) $I_a = j40$, $I_b = 40$, $I_c = 0$ A.

8.4 Find the phase voltages V_{an}, V_{bn}, and V_{cn} whose sequence components are $V_0 = 45\angle 80°$, $V_1 = 90\angle 0°$, $V_2 = 45\angle 90°$ V.

8.5 For the unbalanced three-phase system described by

$$I_a = 10\angle 0° \text{ A},\ I_b = 8\angle -90° \text{ A},\ I_c = 6\angle 150° \text{ A}$$

compute the symmetrical components I_0, I_1, and I_2.

8.6 (a) Given the symmetrical components to be

$$V_0 = 10\angle 0° V,\ V_1 = 80\angle 30° V,\ V_2 = 40\angle -30° V$$

determine the unbalanced phase voltages V_a, V_b, and V_c. (b) Using the results of part (a), calculate the line-to-line voltages V_{ab}, V_{bc}, and V_{ca}. Then determine the symmetrical components of these line-to-line voltages, the symmetrical components of the corresponding phase voltages, and the phase voltages. Compare them with the result of part (a). Comment on why they are different, even though either set results in the same line-to-line voltages.

8.7 One line of a three-phase generator is open-circuited, while the other two are short-circuited to ground. The line currents are $I_a = 0$, $I_b = 1200\angle 150°$, and $I_c = 1200\angle +30°$ A. Find the symmetrical components of these currents. Also find the current into the ground.

8.8 Let an unbalanced, three-phase, Y-connected load (with phase impedances of Z_a, Z_b, and Z_c) be connected to a balanced three-phase supply, resulting in phase voltages of V_a, V_b, and V_c across the corresponding phase impedances.
Choosing V_{ab} as the reference, show that

$$V_{ab,0} = 0; \quad V_{ab,1} = \sqrt{3}V_{a,1}e^{j30°}; \quad V_{ab,2} = \sqrt{3}V_{a,2}e^{-j30°}$$

8.9 Reconsider Problem 8.8 and choosing V_{bc} as the reference, show that

$$V_{bc,0} = 0; \quad V_{bc,1} = -j\sqrt{3}V_{a,1}; \quad V_{bc,2} = j\sqrt{3}V_{a,2}$$

8.10 Given the line-to-ground voltages $V_{ag} = 280\angle 0°$, $V_{bg} = 250\angle -110°$, and $V_{cg} = 290\angle 130°$ volts, calculate (a) the sequence components of the line-to-ground voltages, denoted V_{Lg0}, V_{Lg1} and V_{Lg2}; (b) line-to-line voltages V_{LL0}, V_{LL1}, and V_{LL2}; and (c) sequence components of the line-to-line voltages $V_{LL0} = 0$, V_{LL1}, and V_{LL2}. Also, verify the following general relation: $V_{LL0} = 0$, $V_{LL1} = \sqrt{3}V_{Lg1}\angle +30°$, and $V_{LL2} = \sqrt{3}V_{Lg2}\angle -30°$ volts.

8.11 A balanced Δ-connected load is fed by a three-phase supply for which phase C is open and phase A is carrying a current of $10\angle 0°$ A. Find the symmetrical components of the line currents. (Note that zero-sequence currents are not present for any three-wire system.)

8.12 A Y-connected load bank with a three-phase rating of 500 kVA and 2300 V consists of three identical resistors of 10.58 Ω. The load bank has the following applied voltages: $V_{ab} = 1840\angle 82.8°$, $V_{bc} = 2760\angle -41.4°$, and $V_{ca} = 2300\angle 180°$ V. Determine the symmetrical components of (a) the line-to-line voltages V_{ab0}, V_{ab1}, and V_{ab2}, (b) the line-to-neutral voltages V_{an0}, V_{an1}, and V_{an2}, (c) and the line currents I_{a0}, I_{a1}, and I_{a2}. (Note that the absence of a neutral connection means that zero-sequence currents are not present.)

SECTION 8.2

8.13 The currents in a Δ load are $I_{ab} = 10\angle 0°$, $I_{bc} = 12\angle -90°$, and $I_{ca} = 15\angle 90°$ A. Calculate (a) the sequence components of the Δ-load currents, denoted $I_{\Delta 0}$, $I_{\Delta 1}$, and $I_{\Delta 2}$; (b) the line currents I_a, I_b, and I_c, which feed the Δ load;

and (c) sequence components of the line currents I_{L0}, I_{L1}, and I_{L2}. Also, verify the following general relation: $I_{L0} = 0$, $I_{L1} = \sqrt{3}I_{\Delta 1}\underline{/-30^\circ}$, and $I_{L2} = \sqrt{3}I_{\Delta 2}\underline{/+30^\circ}$.

8.14 The voltages given in Problem 8.10 are applied to a balanced-Y load consisting of $(12 + j16)$ ohms per phase. The load neutral is solidly grounded. Draw the sequence networks and calculate I_0, I_1, and I_2, the sequence components of the line currents. Then calculate the line currents I_a, I_b, and I_c.

8.15 Repeat Problem 8.14 with the load neutral open.

8.16 Repeat Problem 8.14 for a balanced-Δ load consisting of $(12 + j16)$ ohms per phase.

8.17 Repeat Problem 8.14 for the load shown in Example 8.4 (Figure 8.6).

8.18 Perform the indicated matrix multiplications in (8.2.21) and verify the sequence impedances given by (8.2.22) through (8.2.27).

8.19 The following unbalanced line-to-ground voltages are applied to the balanced-Y load shown in Figure 3.3: $V_{ag} = 100\underline{/0^\circ}$, $V_{bg} = 75\underline{/180^\circ}$, and $V_{cg} = 50\underline{/90^\circ}$ volts. The Y load has $Z_Y = 3 + j4\ \Omega$ per phase with neutral impedance $Z_n = j1\ \Omega$. (a) Calculate the line currents I_a, I_b, and I_c without using symmetrical components, (b) Calculate the line currents I_a, I_b, and I_c using symmetrical components. Which method is easier?

8.20 (a) Consider three equal impedances of $(j27)\ \Omega$ connected in Δ. Obtain the sequence networks.
(b) Now, with a mutual impedance of $(j6)\ \Omega$ between each pair of adjacent branches in the Δ-connected load of part (a), how would the sequence networks change?

8.21 The three-phase impedance load shown in Figure 8.7 has the following phase impedance matrix:

$$\boldsymbol{Z}_P = \begin{bmatrix} (5+j10) & 0 & 0 \\ 0 & (5+j10) & 0 \\ 0 & 0 & (5+j10) \end{bmatrix} \Omega$$

Determine the sequence impedance matrix $\boldsymbol{Z}_S$ for this load. Is the load symmetrical?

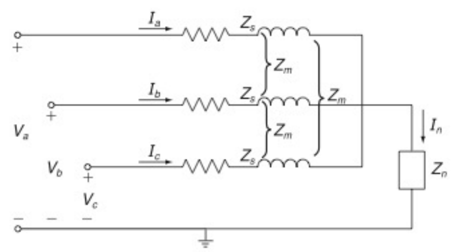
8.22 The three-phase impedance load shown in Figure 8.7 has the following sequence impedance matrix:

$$\boldsymbol{Z}_P = \begin{bmatrix} (6+j10) & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} \Omega$$

Determine the phase impedance matrix Z_P for this load. Is the load symmetrical?

8.23 Consider a three-phase balanced Y-connected load with self and mutual impedances as shown in Figure 8.23. Let the load neutral be grounded through an impedance Z_n. Using Kirchhoff's laws, develop the equations for line-to-neutral voltages, and then determine the elements of the phase impedance matrix. Also find the elements of the corresponding sequence impedance matrix.

FIGURE 8.23
Problem 8.23



8.24 A three-phase balanced voltage source is applied to a balanced Y-connected load with ungrounded neutral. The Y-connected load consists of three mutually coupled reactances, where the reactance of each phase is $j12\ \Omega$, and the mutual coupling between any two phases is $j4\ \Omega$. The line-to-line source voltage is $100\sqrt{3}$ V. Determine the line currents (a) by mesh analysis without using symmetrical components and (b) using symmetrical components.

8.25 A three-phase balanced Y-connected load with series impedances of $(6+j24)\ \Omega$ per phase and mutual impedance between any two phases of $j3\ \Omega$ is supplied by a three-phase unbalanced source with line-to-neutral voltages of $V_{an} = 200\underline{/25^\circ}$, $V_{bn} = 100\underline{/-155^\circ}$, $V_{cn} = 80\underline{/100^\circ}$ V. The load and source neutrals are both solidly grounded. Determine: (a) the load sequence impedance matrix, (b) the symmetrical components of the line-to-neutral voltages, (c) the symmetrical components of the load currents, and (d) the load currents.

SECTION 8.3

8.26 Repeat Problem 8.14 but include balanced three-phase line impedances of $(3 + j4)$ ohms per phase between the source and load.

8.27 Consider the flow of unbalanced currents in the symmetrical three-phase line section with neutral conductor as shown in Figure 8.24. (a) Express the voltage drops across the line conductors given by V_{aa}, V_{bb}, and V_{cc} in terms of line currents, self-impedances defined by $Z_S = Z_{aa} + Z_{nn} - 2Z_{an}$, and mutual impedances defined by $Z_m = Z_{ab} + Z_{nn} = 2Z_{an}$. (b) Show that the sequence components of the voltage drops between the ends of the line section can be written as $V_{aa'0} = Z_0 I_{a0}$, $V_{aa'1} = Z_1 I_{a1}$, and $V_{aa'2} = Z_2 I_{a2}$, where $Z_0 = Z_S + 2Z_m = Z_{aa} + 2Z_{ab} + 3Z_{nn} - 6Z_{an}$ and $Z_1 = Z_2 = Z_S = Z_m = Z_{aa} - Z_{ab}$.

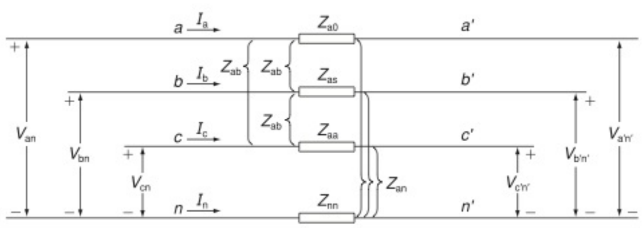


FIGURE 8.24 Problem 8.27

8.28 Let the terminal voltages at the two ends of the line section shown in Figure 8.24 be given by

$V_{an} = (182 + j70)$ kV $\quad V_{an'} = (154 + j28)$ kV

$V_{bn} = (72.24 - j32.62)$ kV $\quad V_{bn'} = (44.24 + j74.62)$ kV

$V_{cn} = (-170.24 + j88.62)$ kV $\quad V_{cn'} = (-198.24 + j46.62)$ kV

The line impedances are given by:

$Z_{aa} = j60\ \Omega \quad Z_{ab} = j20\ \Omega \quad Z_{nn} = j80\ \Omega \quad Z_{an} = 0$

(a) Compute the line currents using symmetrical components. (*Hint:* See Problem 8.27.) (b) Compute the line currents without using symmetrical components.

8.29 A completely transposed three-phase transmission line of 200 km in length has the following symmetrical sequence impedances and sequence admittances:

$Z_1 = Z_2 = j0.5\ \Omega/\text{km}; \quad Z_0 = j2\ \Omega/\text{km}$

$Y_1 = Y_2 = j3 \times 10^{-9}$ s/m; $\quad Y_0 = j1 \times 10^{-9}$ s/m

Set up the nominal π sequence circuits of this medium-length line.

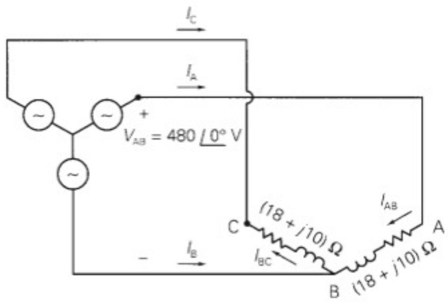
SECTION 8.5

8.30 As shown in Figure 8.25, a balanced three-phase, positive-sequence source with $V_{AB} = 480\underline{/0°}$ volts is applied to an unbalanced Δ load. Note that one leg of the Δ is open. Determine (a) the load currents I_{AB} and I_{BC}; (b) the line currents I_A, I_B, and I_C, which feed the Δ load; and (c) the zero-, positive-, and negative-sequence components of the line currents.

8.31 A balanced Y-connected generator with terminal voltage $V_{bc} = 200\underline{/0°}$ volts is connected to a balanced-Δ load whose impedance is $10\underline{/40°}$ ohms per phase. The line impedance between the source and load is $0.5\underline{/80°}$ ohm for each phase. The generator neutral is grounded through an impedance of $j5$ ohms. The generator sequence impedances are given by $Z_{g0} = j7$, $Z_{g1} = j15$, and $Z_{g2} = j10$ ohms. Draw the sequence networks for this system and determine the sequence components of the line currents.

FIGURE 8.25

Problem 8.30



8.32 In a three-phase system, a synchronous generator supplies power to a 200-volt synchronous motor through a line having an impedance of $0.5\underline{/80°}$ ohm per phase. The motor draws 5 kW at 0.8 p.f. leading and at rated voltage. The neutrals of both the generator and motor are grounded through impedances of $j5$ ohms. The sequence impedances of both machines are $Z_0 = j5$, $Z_1 = j15$, and $Z_2 = j10$ ohms. Draw the sequence networks for this system and find the line-to-line voltage at the generator terminals. Assume balanced three-phase operation.

8.33 Calculate the source currents in Example 8.6 without using symmetrical components. Compare your solution method with that of Example 8.6. Which method is easier?

8.34 A Y-connected synchronous generator rated 20 MVA at 13.8 kV has a positive-sequence reactance of $j2.38\ \Omega$, negative-sequence reactance of $j3.33\ \Omega$, and zero-sequence reactance of $j0.95\ \Omega$. The generator neutral is solidly grounded. With the generator operating unloaded at rated voltage, a so-called single line-to-ground fault occurs at the machine terminals. During this fault, the line-to-ground voltages at the generator terminals are $V_{ag} = 0$, $V_{bg} = 8.071\underline{/-102.25°}$, and $V_{cg} = 8.071\underline{/102.25°}$ kV. Determine the sequence components of the generator fault currents and the generator fault currents. Draw a phasor diagram of the prefault and postfault generator terminal voltages. (*Note:* For this fault, the sequence components of the generator fault currents are all equal to each other.)

8.35 Figure 8.26 shows a single-line diagram of a three-phase, interconnected generator-reactor system, in which the given per-unit reactances are based on the ratings of the individual pieces of equipment. If a three-phase short-circuit occurs at fault point F, obtain the fault MVA and fault current in kA if the prefault busbar line-to-line voltage is 13.2 kV. Choose 100 MVA as the base MVA for the system.

8.36 Consider Figures 8.13 and 8.14 of the text with reference to a Y-connected synchronous generator (grounded through a neutral impedance Z_n) operating at no load. For a line-to-ground fault occurring on phase a of the

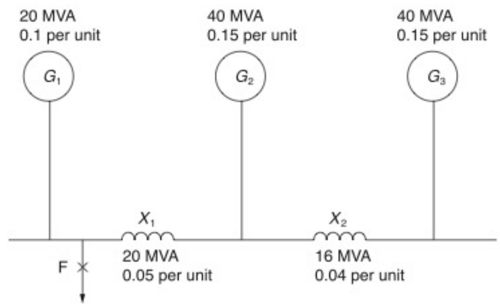


FIGURE 8.26

Oneline diagram for Problem 8.35

generator, list the constraints on the currents and voltages in the phase domain, transform those into the sequence domain, and then obtain a sequence-network representation. Also, find the expression for the fault current in phase *a*.

8.37 Reconsider the synchronous generator of Problem 8.36. Obtain sequence-network representations for the following fault conditions.
(a) A short-circuit between phases *b* and *c*.
(b) A double line-to-ground fault with phases *b* and *c* grounded.

SECTION 8.6

8.38 Three single-phase, two-winding transformers, each rated 450 MVA, 20 kV/288.7 kV, with leakage reactance $X_{eq} = 0.12$ per unit, are connected to form a three-phase bank. The high-voltage windings are connected in Y with a solidly grounded neutral. Draw the per-unit zero-, positive-, and negative-sequence networks if the low-voltage windings are connected (a) in Δ with American standard phase shift or (b) in Y with an open neutral. Use the transformer ratings as base quantities. Winding resistances and exciting current are neglected.

8.39 The leakage reactance of a three-phase, 500-MVA, 345 Y/23 Δ-kV transformer is 0.09 per unit based on its own ratings. The Y winding has a solidly grounded neutral. Draw the sequence networks. Neglect the exciting admittance and assume American standard phase shift.

8.40 Choosing system bases to be 360/24 kV and 100 MVA, redraw the sequence networks for Problem 8.39.

8.41 Draw the zero-sequence reactance diagram for the power system shown in Figure 3.38. The zero-sequence reactance of each generator and of the synchronous motor is 0.05 per unit based on equipment ratings. Generator 2 is grounded through a neutral reactor of 0.06 per unit on a 100-MVA, 18-kV base. The zero-sequence reactance of each transmission line is assumed to be three times its positive-sequence reactance. Use the same base as in Problem 3.41.

8.42 Three identical Y-connected resistors of $1.0\underline{/0^\circ}$ per unit form a load bank that is supplied from the low-voltage Y-side of a Y–Δ transformer. The neutral of the load is not connected to the neutral of the system. The positive- and negative-sequence currents flowing toward the resistive load are given by

$$I_{a,1} = 1\underline{/4.5^\circ} \text{ per unit}; \qquad I_{a,2} = 0.5\underline{/250^\circ} \text{ per unit}$$

and the corresponding voltages on the low-voltage Y-side of the transformer are

$$V_{an,1} = 1\underline{/45^\circ} \text{ per unit (Line-to-neutral voltage base)}$$

$$V_{an,2} = 0.5\underline{/250^\circ} \text{ per unit (Line-to-neutral voltage base)}$$

Determine the line-to-line voltages and the line currents in per unit on the high-voltage side of the transformer. Account for the phase shift.

SECTION 8.7

8.43 Draw the positive-, negative-, and zero-sequence circuits for the transformers shown in Figure 3.34. Include ideal phase-shifting transformers showing phase shifts determined in Problem 3.32. Assume that all windings have the same kVA rating and that the equivalent leakage reactance of any two windings with the third winding open is 0.10 per unit. Neglect the exciting admittance.

8.44 A single-phase three-winding transformer has the following parameters: $Z_1 = Z_2 = Z_3 = 0 + j0.05$, $G_c = 0$, and $B_m = 0.2$ per unit. Three identical transformers, as described, are connected with their primaries in Y (solidly grounded neutral) and with their secondaries and tertiaries in Δ. Draw the per-unit sequence networks of this transformer bank.

SECTION 8.8

8.45 For Problem 8.14, calculate the real and reactive power delivered to the three-phase load.

8.46 A three-phase impedance load consists of a balanced-Δ load in parallel with a balanced-Y load. The impedance of each leg of the Δ load is $Z_\Delta = 6 + j6\ \Omega$, and the impedance of each leg of the Y load is $Z_Y = 2 + j2\ \Omega$. The Y load is grounded through a neutral impedance $Z_n = j1\ \Omega$. Unbalanced line-to-ground source voltages V_{ag}, V_{bg}, and V_{cg} with sequence components $V_0 = 10\underline{/60^\circ}$, $V_1 = 100\underline{/0^\circ}$, and $V_2 = 15\underline{/200^\circ}$ volts are applied to the load. (a) Draw the zero-, positive-, and negative-sequence networks. (b) Determine the complex power delivered to each sequence network. (c) Determine the total complex power delivered to the three-phase load.

8.47 For Problem 8.12, compute the power absorbed by the load using symmetrical components. Then verify the answer by computing directly without using symmetrical components.

8.48 For Problem 8.25, determine the complex power delivered to the load in terms of symmetrical components. Verify the answer by adding up the complex power of each of the three phases.

8.49 Using the voltages of Problem 8.6(a) and the currents of Problem 8.5, compute the complex power dissipated based on (a) phase components and (b) symmetrical components.

CASE STUDIES QUESTIONS

a. What are the components of GIS?

b. What are the typical gas pressures in GIS equipment?

c. What is the environmental concern for SF_6 used in GIS? How is the environmental concern being addressed?

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